

Why the geometry of state space is non-Archimedean

p-Adic Topology

ZP Companion | Version 1.10 | April 2026

This companion explains the ideas in plain language with diagrams and real-world examples. It is not the formal document — every claim here restates a result already proved there. Consult that document for the authoritative mathematics.

Why p-Adic Geometry?

The standard choice for modeling distance — the real number line — is the wrong tool here. The real line is densely ordered: between any two distinct values there is always a third. This means there is no first nonzero step: for any candidate smallest distance ϵ , the value $\epsilon/2$ is smaller and also positive. A framework that requires a definite first departure from null needs a geometry that can enforce one — the real line structurally cannot.

The 2-adic numbers take a different route. As a number system, \mathbb{Q}_2 also has no smallest nonzero element — you can always find something smaller. What changes is the topology: in \mathbb{Q}_2 , zero (0) and every nonzero state live in completely separate clopen regions — entirely disjoint, with no continuous path between them. The gap is not enforced by the number system running out of small values. It is built into the geometry as a hard topological barrier. This is why \mathbb{Q}_2 — the 2-adic number field — is the natural mathematical language for a framework built on AX-B1 (the claim that a state either exists or it does not). Note: this framework makes no claims about physical cosmology. The framework concerns the logical structure of states — it is not a physical theory of how the universe began.

The key property

Mathematical object: 2-adic valuation $v_2(x)$.

What it measures in this framework: depth of binary structure — how many times 2 divides x .

The crucial point: 0 and the first nonzero state lie in separate clopen components of \mathbb{Q}_2 . No continuous path connects them — the geometry enforces the binary gap.

Note on ϵ_0 : the first nonzero state is not a specific number you can look up in \mathbb{Q}_2 — the 2-adic number system has no smallest nonzero element. ϵ_0 is the framework's name for the minimum viable departure from null, defined by structural constraints. The geometry provides the right topology; it does not fix the specific value.

What Is ZP-B Doing?

ZP-B puts the Zero Paradox on a specific geometric foundation: the 2-adic number field \mathbb{Q}_2 . This is not standard Euclidean geometry. In \mathbb{Q}_2 , distance works in a fundamentally different way — one that places

zero at infinite valuation distance from every nonzero element, enforcing a hard structural gap.

The starting point is a single axiom: AX-B1 (Binary Existence) — a state either exists or it does not. From this, together with a minimality principle (MP-1), the document derives that the field must be \mathbb{Q}_2 . The choice of geometry is proven as Theorem T0, not assumed.

Real-world example — Light switch vs. dimmer

AX-B1 says existence is like a light switch: on or off, no dimmer. This is the foundational claim. From "binary only," the 2-adic geometry follows mathematically.

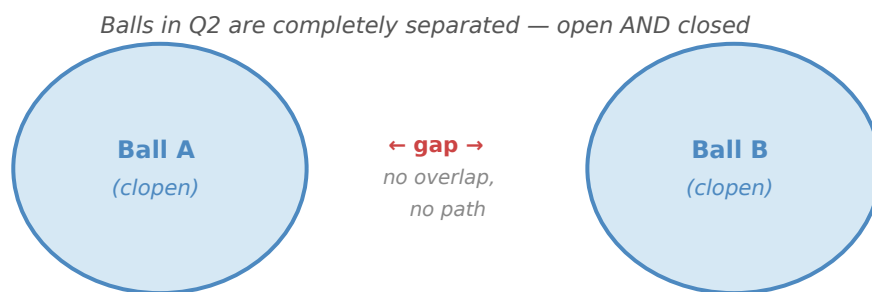
What Does "Non-Archimedean" Mean?

The title uses the term non-Archimedean, which simply means: the usual rule about distance does not apply. In ordinary geometry (Archimedean), if you take enough small steps you can travel any large distance — pile up enough pennies and you reach any amount. A non-Archimedean geometry breaks this: adding more small steps does not necessarily get you further. Distance is measured by a completely different rule — one based on divisibility by 2, not on cumulative size.

The practical consequence: in \mathbb{Q}_2 , two points can be "far apart" even if their numerical difference looks small, and "close together" even if their numerical difference looks large. Closeness is about shared binary structure, not about proximity on the number line.

The Ultrametric: A Different Kind of Distance

In ordinary geometry the triangle inequality says: A-to-C \leq A-to-B plus B-to-C. In \mathbb{Q}_2 a stronger law holds: A-to-C \leq maximum of the two. This is the ultrametric. Every ball is simultaneously open and closed (clopen). There are no gradual boundaries — regions are entirely inside or entirely outside.



Two disjoint balls in \mathbb{Q}_2 . They cannot overlap or gradually merge — completely separated. Every ball is both open and closed.

Real-world example — File folders on a computer

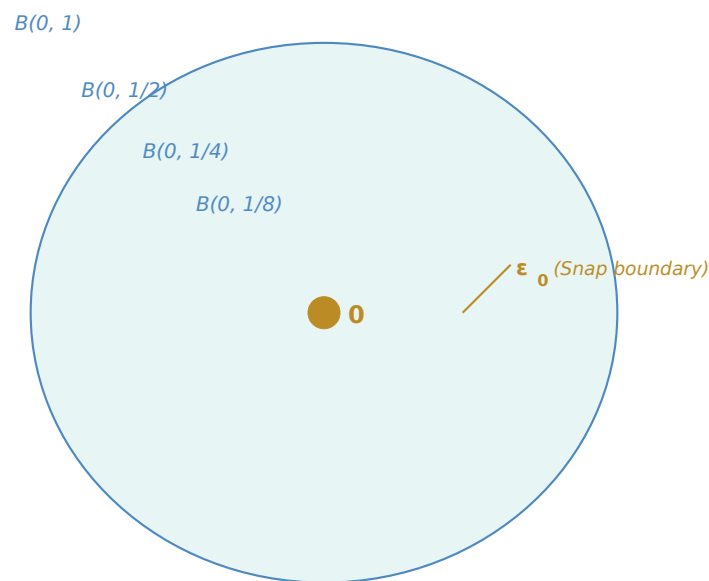
A file is either in a folder or it isn't — there's no "halfway in." Two files in completely different folder trees are maximally separated regardless of how similar their names are. This is ultrametric thinking: location in a hierarchy determines distance, not surface similarity.

The Ball Hierarchy Around Zero

The element $0 \in \mathbb{Q}_2$ is surrounded by nested clopen balls $B(0,1)$, $B(0,1/2)$, $B(0,1/4)$, ... getting smaller and smaller. While sequences of nonzero elements can approach 0 along these nested balls, 0 is clopen-separated from every nonzero element. To understand why, we need the concept of a 2-adic valuation.

The 2-adic valuation of a number counts how many times 2 divides it. For example: $12 = 4 \times 3 = 2^2 \times 3$, so its valuation is 2. The number 7 is not divisible by 2 at all, so its valuation is 0. In \mathbb{Q}_2 , distance between two numbers is determined by their valuation: the more times 2 divides their difference, the closer they are. This is the opposite of ordinary intuition — high divisibility means proximity.

The element 0 has a special status: it is divisible by 2 an unlimited number of times ($0 = 2 \times 0 = 4 \times 0 = \dots$ always). Its valuation is therefore infinite. This means 0 is at infinite "depth" in the hierarchy — no finite sequence of steps from the outside ever reaches it.



Balls in \mathbb{Q}_2 converging on 0 (amber). The ϵ_0 threshold (the first nonzero state in the framework) marks the Snap boundary. 0 is clopen-separated — no continuous path from outside ever reaches it.

Key Result: Total Disconnectedness (T5)

The only connected subsets of \mathbb{Q}_2 are single points. There are no curves, no paths, no continuous structures bridging regions. This is proven from the clopen ball structure — a theorem, not an assumption.

Key Result: The Snap is Topologically Irreversible (C3)

There is no continuous path from any non-zero point back to 0. This follows directly from total disconnectedness (T5). Irreversibility is a corollary of the geometry, not a postulate.

Note: sequences vs. continuous paths

Sequences of nonzero numbers can converge to 0 in \mathbb{Q}_2 . For example, the numbers $(2/3), (2/3)^2, (2/3)^3, \dots$ have 2-adic norms $1/2, 1/4, 1/8, \dots$ approaching 0. This does not contradict C3. A sequence is a list of points; a continuous path is a curve connecting them. Total disconnectedness (T5) rules out continuous paths, not convergent sequences. The snap boundary is a barrier to paths, not to limits.

Real-world example — Crossing a river with no bridge

Imagine a world where rivers have no bridges and no fords. Once you're on one bank, there is literally no continuous path to the other. That's total disconnectedness. 0 is on one side; every non-zero state is on the other. The Snap is the only crossing, and it only goes one way.

Remember: The 2-adic structure — ultrametric, clopen balls, clopen separation of 0 — is fixed by the framework's axioms. ϵ_0 (the Snap threshold) is a structural concept: the minimum nonzero state required by those axioms, not a specific physical constant. The Zero Paradox is a structural argument, not a physical theory of our particular universe.