THE ZERO PARADOX

ZP-C: Information Theory

Version 1.3 | April 2026

Supersedes v1.2 | RP-1 added; D6 extended; T2 rebuilt; branching measure conditionality explicit

Self-contained within information theory and discrete analysis on Q_2. Imports from ZP-B: total disconnectedness (T5), clopen ball hierarchy, AX-B1. Every claim marked as Derived, Axiomatic, Defined, Principle, or Conditional. Version 1.3 changes: (1) RP-1 (Representation Principle) added as explicit bridge between AX-B1 and probabilistic state representations, resolving v1.2 reviewer gap in T1. (2) D6 extended to infinite sequences through the ball hierarchy. (3) T2 rebuilt: finite loop conservation acknowledged; non-conservation proven for infinite sequences approaching 0. (4) T2 status explicitly conditional on the branching measure of D4.

I. Kolmogorov Complexity and the Incompressibility Threshold

1.1 Kolmogorov Complexity

Let x be a binary string of length n. The conditional Kolmogorov complexity is:

$$K(x|n) = \min \{ |p| : U(p, n) = x \}$$

Definition D1 — Incompressibility Threshold
$P_o = \lim_{n\to\infty} K(x|n) / n$
P_o is the algorithmic entropy rate of x. When P_o reaches its maximum, x has exhausted its capacity for self-description.

Remark R1 — Scope of P_o
What P_o establishes (Derived): the point at which x becomes incompressible. A well-defined property of the string.
What P_o does not establish (Axiomatic): that incompressibility causes the Binary Snap. The generative claim is AX-1 in ZP-E. No derivation within information theory produces this claim.

II. State Representations and Informational Work

2.1 The Representation Principle

AX-B1 establishes two ontological states. A bridge principle is required to select the probabilistic form of those states.

Principle RP-1 — Minimum Sufficient Probabilistic Representation
The minimum sufficient probabilistic representation of a binary ontological state is a point-mass (Dirac) distribution over {0,1}: all probability mass is assigned to the value the state occupies, and none to the other value.
Justification: A point-mass distribution is the unique distribution that is (a) faithful — assigns non-zero probability only to the ontologically actual value — and (b) non-redundant — carries no uncertainty about which state is occupied. Any distribution with mass at both values represents partial existence, excluded by AX-B1.
Status: PRINCIPLE. Methodological commitment parallel to MP-1 in ZP-B. Bridges AX-B1 and the probabilistic tools of information theory.

2.2 State Representations Derived from AX-B1 and RP-1

Theorem T1 — State Representations are Uniquely Derived
Given: AX-B1 and RP-1. Claim: $P = (1, 0)$ (Null State) and $Q = (0, 1)$ (First Atomic State)
Step 1: AX-B1 establishes two ontological states: non-existence (value 0) and existence (value 1).
Step 2: RP-1 requires a point-mass distribution for each. The Null State occupies value 0: $P = (1,0)$. The First Atomic State occupies value 1: $Q = (0,1)$.
Step 3: Any $(p, 1-p)$ with $0 < p < 1$ assigns positive mass to both values — partial existence, excluded by AX-B1 and RP-1. P and Q are unique. ✓
Status: DERIVED from AX-B1 and RP-1.

Definition D2 — Jensen-Shannon Divergence

For distributions P and Q with mixture $M = (1/2)(P + Q)$:

$JSD(P\|Q) = (1/2)\, D_{KL}(P\|M) + (1/2)\, D_{KL}(Q\|M)$

JSD is symmetric, bounded in [0,1] bit, and well-defined for distributions with disjoint support.

Theorem T1b — JSD = 1 bit

For $P=(1,0)$ and $Q=(0,1)$ with $M=(1/2,1/2)$:

$D_{KL}(P\|M) = 1$ bit, $D_{KL}(Q\|M) = 1$ bit, $JSD(P\|Q) = 1$ bit

$E = JSD(P\|Q) = 1$ bit [information-theoretic, dimensionless]

Status: Derived from AX-B1 and RP-1 via T1. E is not physical energy in joules. Dimensional bridge belongs to ZP-E as BA-1.

Definition D3 — Dirac Measure δ_0

$\int_\Omega f\, d\delta_0 = f(0)$

δ_0 places unit mass at 0. Compatible with the discrete topology of {0,1} and with the totally disconnected topology of Q_2 (ZP-B T5). δ_0 governs behaviour at $x = 0$; DF (D5) governs behaviour at $x \neq 0$.

Remark R2 — Scope of Hamming Cross-Validation

$d_H(P, Q) = 1$ agrees with $E = 1$ bit. This is a consistency check, not a proof that Hamming distance and JSD are equivalent in general. Both computed by independent methods on AX-B1-derived distributions.

III. The Discrete Surprisal Field on Q_2

Remark R3 — Why the Smooth Embedding Remains Retired

ZP-B T5 establishes Q_2 is totally disconnected. ZP-B T2 establishes every ball is clopen. ZP-B D5 establishes ε_0 as the granular minimum. A totally disconnected, granular space is not a smooth manifold. Importing smooth calculus (gradient, curl) imports a smoothness assumption that directly contradicts ZP-B. The smooth embedding is retired. MO-1 and P1 from v1.1 remain retired.

Definition D4 — Discrete Surprisal Function I and Branching Measure
For $x \in Q_2$ with $x \neq 0$ and $P(x) > 0$: $I(x) = -\log_2 P(x)$
Defined pointwise on $Q_2 \setminus \{0\}$. As $v_2(x) \to +\infty$ (x approaches 0 in the 2-adic metric), $I(x) \to +\infty$.
Branching measure: P on $Q_2 \setminus \{0\}$ is the branching measure induced by the binary ball hierarchy: at each level k, the two sub-balls of $B(0, 2^{-k})$ each receive half the probability mass of their parent. Under this measure, a state x_i with $v_2(x_i) = i$ has $P(x_i) = 2^{-i}$ and $I(x_i) = i$.
Note: The branching measure is defined here, not derived. T2 operates within this measure. Results conditional on the branching measure are so labelled.

Definition D5 — Discrete Surprisal Difference Operator DF
For $x, y \in Q_2 \setminus \{0\}$:
$DF(x, y) = I(y) - I(x) = \log_2[P(x) / P(y)]$
DF is antisymmetric: $DF(x,y) = -DF(y,x)$. No smoothness assumed; defined entirely pointwise.

Definition D6 — Discrete Circulation (Extended in v1.3)
FINITE CASE: A discrete loop $\gamma_n = (x_0, x_1, \ldots, x_n, x_0)$ in $Q_2 \setminus \{0\}$. Its circulation is: $C(DF, \gamma_n) = \Sigma_i DF(x_i, x_{i+1})$
Finite conservation acknowledged: By telescoping, $C(DF, \gamma_n) = I(x_0) - I(x_0) = 0$ for all finite loops. DF is conservative on all finite loops. This is a mathematical fact, not a gap.
INFINITE CASE: An infinite sequence $\sigma = (x_1, x_2, \ldots)$ in $Q_2 \setminus \{0\}$ where $v_2(x_i) = i$ for all $i \geq 1$. Partial circulation: $C_n(DF, \sigma) = I(x_{n+1}) - I(x_1)$ Circulation: $C(DF, \sigma) = \lim_{n\to\infty} C_n(DF, \sigma)$
Relationship to the paradox: Finite loops give $C = 0$; infinite sequences approaching 0 give $C \to \pm\infty$. This is a formal expression of the Zero Paradox: the conservation property holds everywhere except at the approach to the foundational element.

Theorem T2 — DF Exhibits Non-Conservative Behaviour on Infinite Sequences Approaching 0
Claim: DF is conservative on all finite loops (C = 0 by telescoping). On infinite sequences σ through the ball hierarchy approaching 0, C(DF, σ) → +∞.
Step 1 — Finite loops: acknowledged. For any finite loop γ_n, telescoping gives C = 0. Correct and undisputed.
Step 2 — Canonical infinite sequence. Let $\sigma = (x_1, x_2, \ldots)$ where $x_i = 2^i$. Then $v_2(x_i) = i$ and $\lvert x_i \rvert_2 = 2^{-i} \to 0$.
Step 3 — Compute $I(x_i)$. Under the branching measure: $P(x_i) = 2^{-i}$, so $I(x_i) = -\log_2(2^{-i}) = i$.
Step 4 — Partial circulation. $C_n(DF, \sigma) = I(x_{n+1}) - I(x_1) = (n+1) - 1 = n$.
Step 5 — Limit. $C(DF, \sigma) = \lim_{n\to\infty} n = +\infty$. The circulation diverges. ✓
Step 6 — Source of divergence. $I(x_i) = i$ grows without bound as $i \to \infty$, following directly from the 2-adic ball hierarchy and the branching measure. The hierarchy near 0 is unbounded in depth — a structural feature of Q_2 from ZP-B.
Step 7 — No contradiction with finite conservation. Finite loops close; they telescope to zero. Infinite sequences do not close — they approach 0 asymptotically. The limit of partial sums of a non-terminating telescoping series is not constrained to be zero. The two regimes are distinct and consistent.
Status: DERIVED — conditional on the branching measure of D4. Under the branching measure the proof is complete. The branching measure is defined in D4, consistent with AX-B1 and MP-1. Any different measure requires re-examination of T2. OQ-C1 remains closed under this measure. ✓

IV. Validation Status

Component	Status / Notes
RP-1: Representation Principle	Valid — Principle; explicit bridge; resolves v1.2 gap in T1
T1: Distributions from AX-B1 + RP-1	Valid — Derived
T1b: E = JSD = 1 bit	Valid — Derived from AX-B1 + RP-1
δ_0 Dirac measure (D3)	Valid — standard; compatible with discrete Q_2 topology
Smooth embedding, MO-1, P1	Retired — inconsistent with ZP-B; remain retired
D4: I(x) + branching measure	Valid — Defined; branching measure stated explicitly
D5: Difference operator DF	Valid — antisymmetric; no smoothness assumed
D6: Circulation — extended	Valid — finite and infinite cases defined; finite conservation acknowledged
T2: Non-conservation rebuilt	Valid — Derived; conditional on branching measure of D4; conditionality stated in status