ZP-G Illustrated Companion

Structure independent of domain

Category Theory

ZP Companion | Version 1.8 | April 2026

This companion explains the ideas in plain language with diagrams and real-world examples. It is not the formal document — every claim here restates a result already proved in the corresponding technical document. Consult that document for the authoritative mathematics.

Background: What Is a Category?

A category is one of the most general structures in mathematics. It has two ingredients: objects and morphisms (also called arrows).

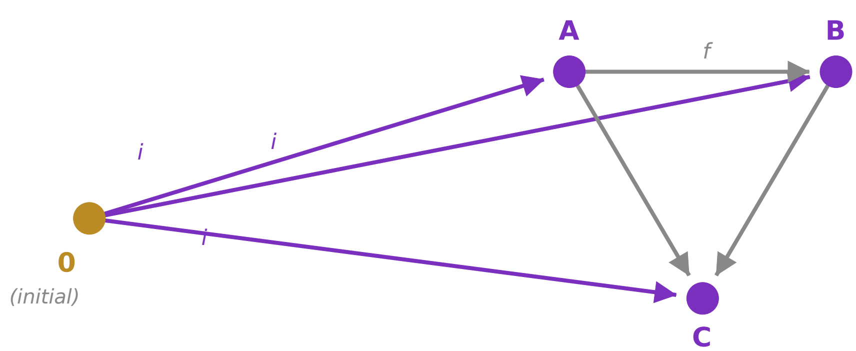
The objects are the things in the category — dots on a diagram. They can be anything: numbers, sets, vector spaces, topological spaces, or the abstract states of the Zero Paradox.

The morphisms are the relationships between objects — arrows from one object to another. A morphism f from object A to object B is written $f: A \to B$. The key rule is that morphisms compose: if $f: A \to B$ and $g: B \to C$, then there is a combined morphism $g \circ f: A \to C$. There is also an identity morphism on every object — the "do nothing" arrow from any object to itself.

That is all a category is. No specific substance is required — just objects, arrows, composition, and identity. The power of the framework is that the same structural theorems apply to any category, regardless of what the objects actually are.

Real-world example — A city road network

Objects = cities. Morphisms = one-way roads. Composition = connecting roads: if there is a road from Boston to New York and a road from New York to Chicago, there is a composed route from Boston to Chicago. The identity morphism is a city road that loops back to itself (a "stay where you are" option). Category theory studies what structures of roads are possible — without caring what the cities actually contain.



A small category with initial object 0 (amber) and objects A, B, C (purple). Purple arrows (i) are the unique morphisms from 0 to each object. Grey arrows are morphisms between non-initial objects. No arrow points back to 0.

Background: What Is a Functor?

A functor is a structure-preserving map between two categories. If C and D are categories, a functor $F: C \to D$ assigns to each object in C an object in D, and to each morphism in C a morphism in D — in a way that respects composition and identity.

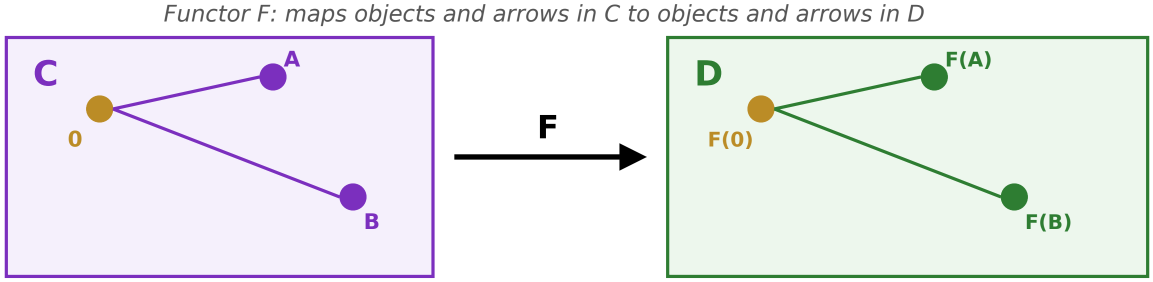
The word "structure-preserving" is the key. If $f: A \to B$ in C, then $F(f): F(A) \to F(B)$ in D. If f and g compose to $g \circ f$ in C, then their images under F compose to $F(g) \circ F(f)$ in D. Functors are the "translations" between mathematical worlds — they carry structure faithfully from one setting to another.

ZP-H (the companion bridge document) constructs four functors from the abstract category C to the four concrete frameworks of the Zero Paradox — lattice algebra, p-adic topology, information theory, and Hilbert space. ZP-G builds the abstract category that those functors will target.

Real-world example — Translation between languages

A functor is like a careful translator. Objects are words; morphisms are grammatical relationships. A good translator preserves structure: if "A causes B" in English, the translation into French should still express "A causes B," not scramble the causal relationship. Functors do the same for mathematical structure.

Functor F: maps objects and arrows in C to objects and arrows in D



Functor $F: C \to D$ maps objects (dots) and morphisms (arrows) from category C (left) to category D (right). The initial object 0 maps to F(0); object A maps to F(A); morphisms are preserved. Structure is carried faithfully across.

What Is ZP-G Doing?

ZP-G constructs a single abstract category C whose structure captures everything essential about the Zero Paradox: there is a privileged starting point (the initial object), all structure flows forward from it, and no morphism ever returns to it. These properties are stated as two axioms within ZP-G — neither is a novel commitment. Both are grounded in structure established in prior layers.

AX-G1 (Initial Object): The category C has an initial object, called 0. An initial object is an object with exactly one morphism to every other object — a universal source. Every other object is "reachable" from 0 by exactly one route. This is not a new assumption: ⊥'s existence as the bottom element of the ZP-A semilattice already guarantees it. ZP-G names it in categorical language.

AX-G2 (Source Asymmetry): No morphism points from any non-initial object back to 0. Once you leave the initial object, you cannot return. This is the categorical expression of irreversibility. Not a new assumption: it follows from antisymmetry of the ZP-A partial order and is independently confirmed by ZP-B C3 (topological irreversibility in $\mathbb{Q}_2$).

The connection to $\bot = \{\bot\}$: ZP-A CC-2 characterizes $\bot$ as a Quine atom — $\bot = \{\bot\}$, meaning $\bot$ is its own only member. A self-containing object has no external interpreter: it IS its own interpretation. In categorical terms, this is exactly what AX-G1 and AX-G2 together express: nothing can reach inside 0 from outside (AX-G2), yet 0 constitutes every object in C through a unique outgoing morphism (T2). No external handle, present in everything — the categorical picture of self-containment.

Remember: 0 here is not the number zero. It is a label for the initial object of the category — the privileged starting point from which all structure originates. The label was chosen because 0 plays the same role as $\bot$ (bottom) in ZP-A, $0 \in \mathbb{Q}_2$ in ZP-B, and $\bot$ in ZP-C.

Key Results

T1: The Initial Object Is Unique

There is exactly one initial object in C (up to a unique isomorphism — a "relabelling" that changes nothing structurally). No two distinct objects can each be a universal source.

T2: Universal Constituent

For every object X in C, there exists a unique morphism from 0 to X. The initial object is a constituent of everything — every object has a unique "origin story" tracing back to 0.

T3: Unreachability

No morphism from any non-initial object ever reaches 0. The initial object is unreachable from outside. This is the categorical expression of AX-G2.

T4: Chains Are Forward-Only

No chain of morphisms starting from a non-initial object returns to 0. Forward movement is the only possibility. The structure of C is strictly directed.

The Informational Asymmetry of 0

ZP-G introduces an informal notion of surprisal for morphisms — how much informational content a transition carries. The definition is structural, not probabilistic: it counts the number of distinct paths from 0 to an object. As the category grows and more objects are added, the number of morphisms departing from 0 grows with them, and the surprisal of leaving 0 increases without bound.

The initial object 0 has a built-in directional asymmetry: outward surprisal (from 0 to anything) grows without bound, while inward surprisal (from anything to 0) is undefined — no such morphism exists (T3). 0 is a one-way origin: reachable as a source, unreachable as a destination.

T6 formalizes this: the surprisal associated with leaving 0 accumulates with each step, and the morphisms pointing back to 0 are absent by AX-G2. 0 is a one-way source of informational content.

T7: The Categorical Zero Paradox

The initial object 0 is simultaneously the universal constituent of C (every object has a unique morphism from 0) and informationally unreachable (no morphism returns to 0 from outside). Presence without return — the categorical statement of the Zero Paradox. This closing theorem of ZP-G shows that the paradox is not an artifact of any one mathematical framework but a structural property of any category satisfying AX-G1 and AX-G2.

What comes next: ZP-H constructs four concrete functors from C to the four domain frameworks of the Zero Paradox, verifying that the abstract categorical structure is faithfully realized in lattice algebra, p-adic topology, information theory, and Hilbert space. See the ZP-H Illustrated Companion for that story.