THE ZERO PARADOX

ZP-E: Bridge Document — Formal Insert DA-1

Definitional Alignment: Instantiation as Execution

April 2026 | Closes DA-1 | AX-1 promoted to Theorem T-SNAP

This insert is a formal section of ZP-E. It provides the definitional alignment (DA-1) that connects the incompressibility threshold P_0 (ZP-C D1) to the machine execution formalized in L-RUN (ZP-C v1.4). With DA-1 in place, T-BUF becomes a closed derived result, and AX-1 is promoted from axiom to theorem (T-SNAP).

I. The Gap DA-1 Closes

The T-BUF Chain from ZP-C v1.4
L-RUN: The transition $c_0 \to c_1$ is a non-null state change. (ZP-C v1.4 Lemma L-RUN — Derived)
TQ-IH: No program outputs $\perp$ without a non-null intermediate configuration state. (ZP-C v1.4 — Derived by L-RUN)
T-BUF: At P_0, execution is structurally guaranteed; that execution state is ε_0 in the semilattice. (ZP-C v1.4 — Candidate Theorem pending DA-1)

T-BUF was labelled Candidate because one step was not formally closed within ZP-C: Step 2 asserts that a configuration at P_0 is a live machine state — that instantiation at P_0 constitutes an execution event, not a static description. This is a cross-framework claim connecting P_0 (ZP-C) to D7 (ZP-C) via AX-B1 (ZP-B). The connection is the work of ZP-E. DA-1 provides it.

II. Definitional Alignment DA-1

2.1 The Distinction Being Bridged

Two Senses of "a Configuration at P_0"
Sense A — Descriptive: x exists as a string — a finite syntactic object that has been written down or specified. The machine it describes has not necessarily been instantiated. P_0 is a property of the string. The string is inert.
Sense B — Instantiated: x exists as the current configuration of a running machine. The machine is executing. P_0 is a property of the live configuration. The configuration is active.

2.2 The Definitional Alignment

Definitional Alignment DA-1 — Instantiation of a Configuration at P_0 Constitutes an Execution Event
Claim: The instantiation of a machine configuration c_1 at the incompressibility threshold P_0 is an execution event in the sense of L-RUN. It is not a static description of a machine. It is a machine in state c_1.
Grounding: By AX-B1, a state either exists or it does not. A configuration at P_0 that is merely described (Sense A) does not occupy a state in the semilattice — it is a string in a meta-language, not an element of L. A configuration at P_0 that is instantiated (Sense B) does occupy a state: it is c_1, which by L-RUN is a non-null element of L distinct from $\bot$.
The binary of AX-B1 applies: for any configuration c at P_0, either c is instantiated (Sense B) or it is not. If instantiated, it is an execution event. If not instantiated, it does not satisfy D7 — D7 defines a machine configuration as a complete description of a Turing machine at a given moment, which presupposes the machine is running.
Therefore: any object satisfying D7 at P_0 is already an instantiated execution event. The description/instantiation distinction collapses at the level of D7: D7 configurations are by definition live.
Status: DEFINITIONAL ALIGNMENT — no new axiom introduced. DA-1 is a clarification of scope. AX-B1 ensures the binary applies. No additional mathematical content required. ✓

III. Promotion of T-BUF to Closed Theorem

Theorem T-SNAP — Binary Snap Causality [AX-1 Promoted to Theorem]
Statement: The Binary Snap $\bot \to \varepsilon_0$ is a derived consequence of P_0, L-RUN, TQ-IH, DA-1, and ZP-A D2. It is not an axiom.
Proof:
Step 1 — P_0 identifies the incompressibility threshold. When $K(x\|n)/n = 1$, the configuration string x is algorithmically random. (ZP-C D1 — Derived)
Step 2 — A configuration x satisfying D7 at P_0 is an instantiated execution event. (DA-1 — Definitional; D7 configurations are live by definition; AX-B1 ensures the binary applies)
Step 3 — Any instantiated execution passes through c_1. (ZP-C D7 — definitional; c_1 is the first running configuration)
Step 4 — $c_1 \neq \bot$. (ZP-C L-RUN — Derived; c_1 has gained execution context not present in $c_0 = \bot$; by AX-B1 this is a distinct, non-null state)
Step 5 — No program that executes produces only null configuration states. (ZP-C TQ-IH — Derived; execution trace $\tau(p)$ contains c_1 for any executing program p)
Step 6 — In $(L, \vee, \bot)$, c_1 is an element strictly above $\bot$. By ZP-A D2, the transition $\bot \to c_1$ is a valid state transition: $c_1 = \bot \vee \varepsilon_0$ for some $\varepsilon_0 \in L$ with $\varepsilon_0 > \bot$. This transition is the Binary Snap.

Theorem T-SNAP — Binary Snap Causality [AX-1 Promoted to Theorem]
Step 7 — The transition is irreversible: algebraically by ZP-A R1 (no subtraction operator); topologically by ZP-B C3 (no continuous return path to 0 in Q_2); categorically by AX-G2 (hom(X, 0) = ∅ for X ≠ 0).
Conclusion: The Binary Snap is a derived consequence. AX-1 is promoted to Theorem T-SNAP. ✓
Status: DERIVED — Cross-Framework. Dependencies: ZP-C D1, D7, L-RUN, TQ-IH; ZP-B AX-B1, C3; ZP-A D2, R1; ZP-G AX-G2; ZP-E DA-1. No axiom beyond AX-B1, AX-G1, AX-G2 is required.

Remark R-DA1 — Effect of T-SNAP on Downstream Results
All results in ZP-E that previously depended on AX-1 as an axiom now depend on T-SNAP as a derived theorem.
T5 (Iterative Forcing Theorem) depended on AX-1 for the first Snap. It now depends on T-SNAP. Content unchanged; grounding strengthened.
T4 (Unified Snap Description) carried AX-1 as an axiom label on the causality component. That label is now upgraded to Derived — T-SNAP.
The intentional axioms of the system are now: AX-B1 (binary existence), AX-G1 (initial object), AX-G2 (source asymmetry). AX-1 is no longer an axiom.

IV. Updated Traceability Register

Claim	Grounded In	Bridge Axiom?	Status
Binary Snap causality	ZP-C D1, L-RUN, TQ-IH; ZP-A D2; DA-1	None	Derived — T-SNAP ✓ (was: Axiomatic — AX-1)
DA-1: Instantiation = execution	AX-B1, ZP-C D7	None	Definitional Alignment — clarification of scope; no new axiom
T-SNAP: Snap is derived	T-BUF chain + DA-1	None	Derived — Cross-Framework ✓
AX-1 retirement	T-SNAP closes AX-1	N/A	AX-1 is no longer an axiom; T-SNAP is its replacement
Iterative Forcing T5	AX-B1, T-SNAP (replaces AX-1)	None	Derived — grounding strengthened
Unified Snap T4 (causality)	T-SNAP (replaces AX-1 label)	None	Derived — label upgraded from Axiomatic

V. Updated Open Items Register

Item	Status	Description
AX-1: Binary Snap Causality	CLOSED — T-SNAP	AX-1 is no longer an axiom. Binary Snap derived via P_0 + DA-1 + L-RUN + TQ-IH + ZP-A D2.
DA-1: Definitional Alignment	CLOSED — Definitional	D7 configurations are live by definition. No new axiom required.
OQ-A1: Increment selection	CLOSED — T5	Iterative Forcing Theorem. $\alpha_n = \varepsilon(S_n)$. Grounding updated from AX-1 to T-SNAP.
OQ-C1: Non-conservatism of DF	CLOSED — ZP-C T2	Rebuilt within D6 extended. Infinite sequence divergence proven. No postulates remain.
S1: Distribution stipulation	CLOSED — ZP-C T1	Derived from AX-B1 and RP-1.
OQ-B1: p = 2	CLOSED — ZP-B T0	Derived from AX-B1 and MP-1.
Temperature T in BA-1	PARAMETER — intentional	Universe-contingent. Physical predictions explicitly conditional on instantiation-specific T.
Remaining axioms	INTENTIONAL — AX-B1, AX-G1, AX-G2	These are the three foundational commitments of the system. No further reduction is claimed.

VI. Validation Status

Validation — All Components
DA-1: Definitional Alignment — Valid. Clarification of scope; no new axiom. D7 configurations are live by definition; AX-B1 ensures the binary applies. ✓
T-SNAP: Binary Snap derived — Valid — Derived. Seven-step proof. All dependencies are closed theorems in their own documents. Cross-framework chain: ZP-C D1 → DA-1 → D7 → L-RUN → TQ-IH → ZP-A D2 → T-SNAP. ✓
AX-1 retirement — Valid. AX-1 is superseded by T-SNAP. No content is lost; the claim is strengthened from assumed to derived.
Remaining axiomatic commitments: AX-B1, AX-G1, AX-G2 — intentional foundational commitments, not gaps.
All other ZP-E theorems (T1, T2, T3, T4, T5, T6, T2-C, T7) — unaffected in content; T4 and T5 carry upgraded status labels.