

# THE ZERO PARADOX

## ZP-G: Category Theory

Version 1.6 | May 2026

Supersedes v1.4 | v1.5: Lean scope disclosure for T6-b and T6-c strengthened — Lean proofs verify only that a  $\mathbb{N}$ -valued function is  $\geq 0$  (`Nat.zero_le _`), which is trivially true by type for any such function and says nothing about Kolmogorov complexity. T6-b strict inequality ( $K > 0$  for distinct objects) and T6-c subadditivity have no Lean proofs. Status lines and validation table updated accordingly. | v1.4: Lean scope note added after T6-c — K-specific AIT content outside ZPSurprisal skeleton identified | All prior results unchanged

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This document is self-contained within category theory, with one explicit import from ZP-C: the conditional Kolmogorov complexity  $K(x|n)$  and the coding theorem connecting it to Shannon entropy. That import is named and labelled; it replaces the v1.0 Bridge Axiom BA-G1 (Leinster categorical entropy characterization) with a native derivation. All other content from ZP-A, ZP-B, ZP-D, and ZP-E remains excluded from this document. Cross-framework connections are deferred to ZP-H.

Honest labelling is the governing discipline. Every claim is marked as Axiom, Definition, Derived, Import, Design Commitment, or Remark. Nothing slides between categories.

Version 1.1 changes from v1.0: (1) D7 replaced by D7' (conditional Kolmogorov complexity  $K(B|A)$  as native categorical surprisal). (2) Import I-KC added:  $K(x|y)$  and the Kolmogorov coding theorem imported from ZP-C as a named dependency. (3) BA-G1 (Leinster Bridge Axiom) demoted to Remark R-BA: a compatibility result, no longer a theorem premise. (4) T6 rebuilt on D7'; proof is now self-contained within ZP-G plus the named ZP-C import. OQ-G1 closed. (5) All theorems from v1.0 that did not depend on BA-G1 (T1 through T5, T7) are unchanged in statement and proof.

Version 1.2 changes from v1.1: Theorem/Proposition/Lemma hierarchy applied throughout. T1 relabelled Proposition (subsidiary uniqueness result). T2, T3 relabelled Lemma (stepping-stone results for T6/T7). T4, T5 relabelled Proposition (derived but subsidiary). T6-a, T6-b relabelled Lemma (helpers for T6). T6-c relabelled Proposition (supports T6 but not the central claim). T6 and T7 remain Theorems — central claims of their sections. All statements and proofs are unchanged.

Version 1.3 changes from v1.2: Remark R2 added (Categorical Expression of Self-Containment). Connects initial object structure (T2 + AX-G2) to ZP-A CC-2 ( $\perp = \{\perp\}$ ). All prior results, axioms, and definitions unchanged.

Version 1.4 changes from v1.3: Lean scope note added after T6-c — T6-b strict inequality and T6-c subadditivity are K-specific AIT content outside the ZPSurprisal skeleton; Lean proofs reduce to `Nat.zero_le _` (non-negativity by type). T6-b and T6-c statements and proofs unchanged.

Version 1.6 changes from v1.5: D7' well-definedness note corrected — "all structural claims below are invariant under this additive constant" replaced with explicit scoping: invariance holds because of what ZP-G chooses to claim (finite/zero/undefined), not as a general property of K-complexity.

Version 1.5 changes from v1.4: Lean scope disclosure strengthened. The v1.4 note correctly identified the limitation but did not make the consequence fully explicit. v1.5 states it plainly: the Lean proofs for T6-b and T6-c verify nothing about Kolmogorov complexity. `Nat.zero_le _` is trivially true for any  $\mathbb{N}$ -valued function, regardless of mathematical content. T6-b (strict inequality) and T6-c (subadditivity) are not Lean-verified. T6-b and T6-c status lines updated from "Derived — ✓" to "Derived (PDF-level); not Lean-verified". Validation table updated accordingly.

Illustrated Companion: A paired ZP-G Illustrated Companion provides concrete examples and visual intuitions for the results here. Examples are kept separate from the formal layers to distinguish illustrative

material from proofs.

Note on sequencing: The Zero Paradox framework labels its layers A through H, intentionally omitting F. ZP-G follows ZP-E directly; there is no missing document.

## I. Categorical Primitives

Definitions D1 through D6 and results T1 through T5 are unchanged from v1.0. They are reproduced here for completeness and self-reference.

### 1.1 The Definition of a Category

#### Definition D1 — Category

Status: Definition — foundational

A category  $C$  consists of:

(i) Objects: A collection  $\text{ob}(C)$ , written  $A, B, X, 0, \dots$

(ii) Morphisms: For each ordered pair  $(A, B)$ , a collection  $\text{hom}(A, B)$  of morphisms, written  $f: A \rightarrow B$ .

(iii) Composition:  $\circ: \text{hom}(B, C) \times \text{hom}(A, B) \rightarrow \text{hom}(A, C)$ , written  $g \circ f$ .

(iv) Identity: For each  $A$ , a morphism  $\text{id}_A: A \rightarrow A$ .

Associativity:  $h \circ (g \circ f) = (h \circ g) \circ f$ .

Unit laws:  $\text{id}_B \circ f = f = f \circ \text{id}_A$ .

#### Definition D2 — Morphism Uniqueness Notation

Status: Definition

A morphism  $f: A \rightarrow B$  is unique if for any  $g, h: A \rightarrow B$ ,  $g = h$ . Written:  $\exists! f: A \rightarrow B$ .

### 1.2 Initial and Terminal Objects

#### Definition D3 — Initial Object

Status: Definition — load-bearing

An object  $0 \in \text{ob}(C)$  is initial if for every  $X \in \text{ob}(C)$ , there exists a unique morphism  $\iota_X: 0 \rightarrow X$ .

#### Proposition T1 — Uniqueness of the Initial Object

Status: Derived — standard category theory [unchanged from v1.0]

If  $0$  and  $0'$  are both initial in  $C$ ,  $\exists!$  isomorphism  $0 \cong 0'$ . The initial object is unique up to unique isomorphism.

### Proposition T1 — Uniqueness of the Initial Object

Proof:  $\exists! f: 0 \rightarrow 0'$  and  $\exists! g: 0' \rightarrow 0$ . Initiality forces  $g \circ f = \text{id}_0$  and  $f \circ g = \text{id}_{0'}$ . Therefore  $f$  is a unique isomorphism. ✓

### Definition D4 — Terminal Object

Status: Definition — defined for exclusion

An object  $1 \in \text{ob}(C)$  is terminal if for every  $X \in \text{ob}(C)$ ,  $\exists!$  morphism  $\tau_X: X \rightarrow 1$ . The Zero Paradox framework is not a zero object category ( $0 \not\cong 1$ ).

## II. The Foundational Axioms of ZP-G

### Axiom AX-G1 — Asymmetry Axiom

Status: Axiom — foundational structural commitment [unchanged from v1.0]

The category  $C$  possesses an initial object  $0$  and no terminal object.

Formally:  $\exists 0 \in \text{ob}(C)$  satisfying D3.  $\neg \exists 1 \in \text{ob}(C)$  satisfying D4.

Correspondence: In ZP-A: join-semilattice without  $\top$  and without  $\wedge$ . In ZP-B:  $Q_2$  has no element to which all paths converge. The present axiom is the categorical generalization.

### Axiom AX-G2 — Source Asymmetry

Status: Axiom — foundational [unchanged from v1.0]

For any non-initial object  $X \neq 0$ :  $\text{hom}(X, 0) = \emptyset$ .

Motivation: The categorical expression of irreversibility. Morphisms  $\iota_X: 0 \rightarrow X$  exist for all  $X$ . Their reversal does not exist. AX-G2 is consistent with AX-G1 but not derivable from it.

## III. Universal Constituent and Unreachability

### Lemma T2 — Universal Constituent

Status: Derived — from D3 and AX-G1 [unchanged from v1.0]

For every  $X \in \text{ob}(C)$ ,  $\exists! \iota_X: 0 \rightarrow X$ . The initial object  $0$  is the universal categorical source.

Proof: Immediate from D3 and AX-G1. ✓

### Lemma T3 — Unreachability of 0

Status: Derived — from AX-G2 [unchanged from v1.0]

For any  $X \neq 0$ :  $\text{hom}(X, 0) = \emptyset$ . The initial object  $0$  is unreachable from any non-initial object.

### Lemma T3 — Unreachability of 0

Proof: Direct from AX-G2. ✓

### Remark R1 — Structural Inversion — The Categorical Zero Paradox

Status: Remark [unchanged from v1.0]

T2 and T3 together constitute the categorical Zero Paradox. 0 reaches every object (T2); no non-initial object reaches 0 (T3). This is not a logical contradiction. It is a structural inversion: the unique universal source is the unique object with no incoming non-trivial morphisms.

### Remark R2 — Categorical Expression of Self-Containment

Status: Remark — connecting note to ZP-A CC-2 [new in v1.3]

R1 frames the structural inversion of 0. This remark connects that structure to ZP-A CC-2:  $\perp = \{\perp\}$ . The null state is its own extension — a Quine atom. A self-containing object has no external interpreter by structure; it IS its own interpretation.

In categorical terms, this corresponds to two conditions together: (1) AX-G2:  $\text{hom}(X, 0) = \emptyset$  for all  $X \neq 0$  — no morphism can reach inside 0 from outside; and (2) T2:  $\exists! \iota_X: 0 \rightarrow X$  for all  $X$  — 0 is structurally present in every object. Together these are the categorical image of undifferentiated self-containment: unreachable from without, yet constitutive of everything.

0 "points in all directions" (T2) because it is the undifferentiated ground from which all differentiation proceeds — not because it selects a direction. The uniqueness of each  $\iota_X$  is not a choice among alternatives; it is the absence of internal structure that would allow differentiation among morphisms.

This remark bridges ZP-G to ZP-A CC-2. The formal correspondence between the categorical initial object structure and the set-theoretic Quine atom  $\perp = \{\perp\}$  is made explicit in ZP-H.

## IV. Monotone Structure and the Additive Ontology

### Definition D5 — Morphism Chain

Status: Definition [unchanged from v1.0]

A morphism chain of length n from 0 is a sequence:

$$0 = X_0 \rightarrow X_1 \rightarrow \dots \rightarrow X_n$$

where each  $X_k \rightarrow X_{k+1}$  is a morphism in C.

### Proposition T4 — Chains are Forward-Only

Status: Derived — from AX-G2 [unchanged from v1.0]

No morphism chain from 0 can return to 0 through non-initial objects.

### Proposition T4 — Chains are Forward-Only

Proof: A return morphism  $X_n \rightarrow 0$  for  $X_n \neq 0$  would contradict AX-G2. ✓

### Definition D6 — Functor

Status: Definition — standard category theory [unchanged from v1.0]

A functor  $F: C \rightarrow D$  consists of an object map  $F: \text{ob}(C) \rightarrow \text{ob}(D)$  and a morphism map  $F: \text{hom}_C(A, B) \rightarrow \text{hom}_D(F(A), F(B))$ , preserving composition and identity.

### Proposition T5 — Functors Preserve Initial Objects

Status: Derived — [OQ-G2 closed in ZP-H T-H1; unchanged from v1.0]

For each instantiation functor  $F \in \{F_A, F_B, F_C, F_D\}$ ,  $F(0)$  is an initial object in the codomain. Verified by direct universal property check in ZP-H T-H1. ✓

## V. The Kolmogorov Import from ZP-C

This section contains the single import from outside category theory that ZP-G v1.1 requires. It is named, scoped, and labelled explicitly. It replaces BA-G1 (the Leinster Bridge Axiom from v1.0) as the informational foundation of D7'.

### Import I-KC — Conditional Kolmogorov Complexity from ZP-C

Status: Import — from ZP-C D1 and standard algorithmic information theory

What is imported: The conditional Kolmogorov complexity  $K(x|y)$ , defined as the length of the shortest program  $p$  such that  $U(p, y) = x$ , where  $U$  is a fixed universal Turing machine:

$$K(x|y) = \min \{ |p| : U(p, y) = x \}$$

Key property — The Coding Theorem: For any computable probability measure  $P$ , Kolmogorov complexity and Shannon entropy are related up to an additive constant  $c$  (depending only on  $U$ , not on  $x$  or  $y$ ):

$$K(x|y) \approx -\log_2 P(x|y) + O(c)$$

The coding theorem is a standard result of algorithmic information theory (Li and Vitanyi, An Introduction to Kolmogorov Complexity and Its Applications). It is not derived within ZP-G. It is imported as a named result.

Scope of import: I-KC imports  $K(x|y)$  and the coding theorem only. No other content from ZP-C is imported into ZP-G. The Kolmogorov complexity machinery is already present in ZP-C D1 (incompressibility threshold  $P_0$ ), so I-KC introduces no new external dependency into the overall Zero Paradox framework — it only introduces a dependency within ZP-G specifically.

Computability note:  $K(x|y)$  is not computable in general (it is approximable from above by standard results). This is not a defect for the present purposes: the framework requires that  $K(x|y)$  be well-defined, not that it be computable. The ontological claims of ZP-G do not depend on computability.

## Import I-KC — Conditional Kolmogorov Complexity from ZP-C

Status: This is an import, not a bridge axiom. A bridge axiom is a claim that cannot be derived from either side and must be assumed.  $K(x|y)$  is a defined mathematical object with a complete internal theory. I-KC is a decision to use that object within ZP-G, not an assumption about it.

## VI. Categorical Information Theory [Rebuilt in v1.1]

### 6.1 Native Categorical Surprisal — Definition D7'

Version 1.0 defined categorical surprisal via the Shannon entropy functor  $H$  imported through BA-G1. Version 1.1 replaces this with conditional Kolmogorov complexity, imported via I-KC. The definition is native to the morphism structure of  $C$ .

#### Definition D7' — Native Categorical Surprisal

Status: Definition — from D5, I-KC [replaces D7 from v1.0]

Let  $f: A \rightarrow B$  be a morphism in  $C$ . Represent  $A$  and  $B$  as binary strings  $x_A$  and  $x_B$  via any injective encoding consistent with the morphism structure of  $C$ . The categorical surprisal of  $f$  is:

$$I(f) = K(x_B | x_A)$$

the conditional Kolmogorov complexity of the target given the source.

Interpretation:  $I(f)$  measures the minimum description length of  $B$  given knowledge of  $A$ . It is the irreducible informational content added by the transition  $f: A \rightarrow B$ , independent of any probability distribution.

Well-definedness:  $K(x_B | x_A)$  depends on the encoding of objects as strings. Different encodings yield values differing by at most an additive constant  $c$  (the coding theorem constant of I-KC). All claims in ZP-G are stated in terms of whether  $I(f)$  is finite, zero, or undefined — qualitative properties that are invariant under any additive constant. This invariance is a consequence of what ZP-G chooses to claim, not a general property of  $K$ -complexity (additive constants can matter for precise  $K$ -complexity comparisons in AIT).

Relationship to v1.0 D7: By the coding theorem (I-KC),  $K(x_B | x_A) \approx -\log_2 P(x_B | x_A) + O(c)$  for any computable measure  $P$ . Therefore D7' and D7 are equivalent up to  $O(c)$ . The choice of D7' over D7 is not a change in what is being measured; it is a change in how the measure is defined — natively versus via import.

### 6.2 Properties of the Native Surprisal

#### Lemma T6-a — Surprisal of the Identity Morphism is Zero

Status: Derived — from D7' and I-KC

Claim:  $I(\text{id}_A) = K(x_A | x_A) = 0$  up to the additive constant  $c$ .

Proof: The shortest program producing  $x_A$  given  $x_A$  is the empty program (output the input). Therefore  $K(x_A | x_A) = 0$  up to  $c$ . The identity morphism adds no informational content. ✓

### Lemma T6-b — Surprisal is Non-Negative for Forward Morphisms

Status: Derived — PDF-level (from D5, D7', AX-G2); NOT Lean-verified — see Lean scope remark below

Claim: For any morphism  $f: A \rightarrow B$  in a forward morphism chain from 0,  $I(f) \geq 0$  up to  $c$ , with strict inequality when  $A \neq B$ .

Proof:  $K(x_B|x_A) \geq 0$  by definition (program length is non-negative). Strict inequality holds when  $x_B$  cannot be computed from  $x_A$  by the empty program — i.e., when  $A$  and  $B$  are distinct objects encoding distinct states. In a forward morphism chain (D5), each step adds content by the additive ontology (AX-G2). ✓

### Proposition T6-c — Surprisal Accumulates Along Chains

Status: Derived — PDF-level (from D5, T6-b, subadditivity of  $K$ ); NOT Lean-verified — see Lean scope remark below

Claim: For a morphism chain  $0 = X_0 \rightarrow X_1 \rightarrow \dots \rightarrow X_n$ , the total surprisal  $\sum I(X_k \rightarrow X_{k+1}) \geq 0$ , with monotone accumulation as  $n$  increases.

Proof: By subadditivity of Kolmogorov complexity:  $K(x_n|x_0) \leq \sum K(x_{k+1}|x_k) + O(n \cdot c)$ . Each term is  $\geq 0$  by T6-b. Adding distinct objects strictly increases the total. ✓

### Remark — Lean Scope of T6-b and T6-c

Status: Scope note [strengthened v1.5] — T6-b and T6-c are NOT Lean-verified for their mathematical claims

T6-b and T6-c are not Lean-verified. The ZPG.lean proofs for these results compile without error, but they verify nothing about Kolmogorov complexity. The ZPSurprisal typeclass defines `surp : hom → ℕ` (surprisal as a natural number) and the proofs reduce to `Nat.zero_le _`, which states that any natural number is  $\geq 0$ . This is trivially true by type for any  $\mathbb{N}$ -valued function, regardless of its mathematical content. A compiling Lean proof here does not mean the K-theoretic claims have been verified.

What the Lean proofs do NOT establish: (1) T6-b strict inequality —  $K(x_B|x_A) > 0$  when  $A \not\cong B$  (distinct objects cannot have zero description length). (2) T6-c subadditivity —  $K(x_n|x_0) \leq \sum K(x_{k+1}|x_k) + O(n \cdot c)$  (total surprisal along a chain). These are standard and correct results from algorithmic information theory, but they require a K-formalization that does not exist in Lean 4 / Mathlib.

What IS Lean-verified: T6-a (identity morphism has zero surprisal — from `surp_id`), T6 Part II (inward surprisal is undefined — from AX-G2 and the absence of morphisms to 0), and T7 insofar as it depends on Parts II, III, V, VI. T6 Part I (outward accumulation) and T7 Part IV rely on T6-b and T6-c and are therefore also PDF-level only.

Readers citing "the Lean-verified ZP-G framework" should note this boundary. The PDF-level arguments for T6-b and T6-c are mathematically valid — these are standard K-theoretic results (Li and Vitanyi). The gap is Lean scope, not mathematical correctness.

## VII. The Informational Singularity of 0 [Rebuilt in v1.1]

This is the central theorem of the information-theoretic section. In v1.0, it depended on BA-G1. In v1.1, it is proved from D7' and AX-G2 alone, with I-KC as the only external dependency.

## Theorem T6 — Informational Singularity of 0

Status: Derived — from AX-G2, D7', I-KC [rebuilt from v1.0, OQ-G1 closed]

Setup: Let 0 be the initial object of C (AX-G1). Let  $I(f) = K(x_B|x_A)$  be the categorical surprisal (D7'). Let I-KC provide the Kolmogorov framework.

Part I — Outward surprisal accumulates (from T6-b, T6-c): For any morphism chain  $0 = X_0 \rightarrow \dots \rightarrow X_n, \sum I(X_k \rightarrow X_{k+1}) \geq 0$ , with strict accumulation as n increases. ✓

Part II — Inward surprisal is undefined (from AX-G2): For any  $X \neq 0$ ,  $\text{hom}(X, 0) = \emptyset$  (AX-G2). Therefore D7' cannot be applied to any morphism  $f: X \rightarrow 0$  from outside 0 — no such morphism exists.  $I(X \rightarrow 0)$  is undefined not because K diverges to infinity, but because there is no morphism to apply D7' to. The undefined-domain condition is strictly stronger than divergence. ✓

Part III — The singularity: 0 is the unique object in C for which outward surprisal is defined and accumulates (Part I) while inward surprisal is undefined by absence of morphisms (Part II). This is the informational singularity: the initial object is informationally accessible in the outward direction and categorically inaccessible in the inward direction. The singularity is structural, not numerical. It does not require K to diverge — it requires only AX-G2 and D7'. ✓

Comparison with ZP-C (to be reconciled in ZP-H T-H2): ZP-C establishes that the discrete surprisal DF diverges (numerically, to  $\infty$ ) along infinite sequences approaching 0 in  $Q_2$ . T6 Part II establishes that  $I(X \rightarrow 0)$  is undefined (domain-absent) for any  $X \neq 0$  in C. These are compatible: undefined is stronger than infinite. ZP-H T-H2 proves they describe the same obstruction under the functor  $F_C$ .

Status: DERIVED. Depends on AX-G1, AX-G2, D3, D5, D7', I-KC, T6-a, T6-b, T6-c. OQ-G1 is closed. BA-G1 is no longer a premise of T6. ✓

## 7.1 Compatibility with Shannon Entropy — BA-G1 Demoted to Remark

### Remark R-BA — Compatibility of D7' with the Shannon Entropy Functor

Status: Remark — BA-G1 demoted from Bridge Axiom [v1.0] to Compatibility Remark [v1.1]

Version 1.0 introduced BA-G1 as a bridge axiom: it imported Leinster's categorical characterization of Shannon entropy (naturality, maximality, chain rule) to define the surprisal functor. BA-G1 was the only bridge axiom in ZP-G v1.0 and was the source of OQ-G1.

In v1.1, BA-G1 is no longer a premise of any theorem. It is retained here as a compatibility remark: the coding theorem (I-KC) guarantees that D7' and the Shannon functor of BA-G1 are equivalent up to an additive constant c. Specifically:

$$K(x_B|x_A) \approx H(F(B)) - H(F(A)) + O(c)$$

for any computable probability measure P consistent with the morphism structure of C. This means all quantitative results that v1.0 derived from BA-G1 remain valid under D7' — they differ only by the additive constant c, which does not affect any structural (finite/zero/undefined) claim.

BA-G1 is not false. It is not retired. It is now a derived compatibility result rather than an assumed premise. Any reader who finds the Shannon characterization more intuitive than Kolmogorov complexity may use BA-G1 as an equivalent formulation, knowing that I-KC and the coding theorem connect them.

## VIII. The Categorical Zero Paradox — Formal Statement

Theorem T7 is the closing theorem of ZP-G. Its statement is unchanged from v1.0. Its proof is strengthened: Part IV (informational singularity) now rests on T6 as rebuilt in v1.1, which does not depend on BA-G1.

Theorem T7 — The Categorical Zero Paradox	
Status: Derived — Closing Theorem [Part IV strengthened in v1.1]	
Setup: Let C satisfy AX-G1 and AX-G2. Let I be the categorical surprisal from D7'. Let I-KC provide the Kolmogorov framework.	
Part I — Universal Constituent (T2): $\forall X \in \text{ob}(C), \exists! \iota_X: 0 \rightarrow X$ . The initial object 0 is the universal categorical source.	
Part II — Unreachability (T3): $\forall X \neq 0, \text{hom}(X, 0) = \emptyset$ . No non-initial object reaches 0.	
Part III — Forward Irreversibility (T4): No morphism chain from 0 can return to 0 through non-initial objects.	
Part IV — Informational Singularity (T6, rebuilt): $I(X \rightarrow 0)$ is undefined for all $X \neq 0$ (no such morphism exists, AX-G2). Outward surprisal from 0 accumulates along any morphism chain (T6-b, T6-c). 0 is an informational singularity: undefined inward, accumulating outward. This part no longer depends on BA-G1.	
Part V — The Structural Inversion: Parts I and II together constitute the paradox. 0 is the unique universal source of all objects, and simultaneously the unique object unreachable from outside. The foundation is the one object the morphism machinery cannot return to.	
Part VI — Resolution: The paradox is not a logical contradiction. It is a structural inversion. The correct tools for characterizing 0 are the universal property (D3) and D7' applied to outward morphisms from 0. Under these tools, 0 is fully characterized. The paradox is the precise boundary between what can reach 0 and what cannot.	
Status: DERIVED — Closing Theorem. Depends on D3, D5, D7', AX-G1, AX-G2, I-KC, T2, T3, T4, T6. BA-G1 is not a dependency. ✓	

## IX. Open Items Register for ZP-G v1.5

Item	Status	Description
OQ-G1	Closed — D7', T6	Native categorical derivation of surprisal without importing Shannon entropy. Closed by replacing D7 with D7' (conditional Kolmogorov complexity $K(B A)$ ). BA-G1 demoted from Bridge Axiom to Compatibility Remark R-BA. The single remaining external dependency is I-KC (Kolmogorov framework from ZP-C), which is an import, not a bridge axiom.
OQ-G2	Closed — ZP-H T-H1	Left adjoint verification for instantiation functors. Resolved in ZP-H v1.0 by direct universal property verification for each functor.
OQ-G3	Closed — ZP-H C-H1 through C-H4	Explicit construction of four instantiation functors. Resolved in ZP-H v1.0.

Item	Status	Description
OQ-G4	Closed — ZP-H T-H2	Reconciliation of categorical and ZP-C singularity characterizations. Resolved in ZP-H v1.0. Undefined domain (ZP-G) and infinite accumulation (ZP-C) shown to be the same obstruction under the functor $F_C$ .
I-KC	Import — named dependency	Conditional Kolmogorov complexity $K(x y)$ and the coding theorem, imported from ZP-C D1 and standard algorithmic information theory. This is an import, not a bridge axiom: $K(x y)$ is a fully defined mathematical object. ZP-G is no longer purely categorical; this dependency is explicitly stated.
AX-G1	Axiom — intentional	Asymmetry: initial object 0, no terminal object. Foundational structural commitment. Not a gap.
AX-G2	Axiom — intentional	Source asymmetry: $\text{hom}(X, 0) = \emptyset$ for $X \neq 0$ . Foundational irreversibility commitment. Not a gap.
R-BA	Remark — BA-G1 demoted	Leinster Shannon entropy characterization is now a compatibility remark, not a bridge axiom premise. Derivable from D7' and I-KC via the coding theorem. Not a gap.

## X. Validation Status

Component	Status / Notes
D1: Category	Valid — Definition. Standard; foundational. Unchanged.
D2: Uniqueness notation	Valid — Definition. Unchanged.
D3: Initial object	Valid — Definition. Load-bearing for T2, T7. Unchanged.
D4: Terminal object	Valid — Definition. Defined for exclusion. AX-G1 prohibits it. Unchanged.
D5: Morphism chain	Valid — Definition. Native to C. Unchanged.
D6: Functor	Valid — Definition. Standard. Unchanged.
D7': Native categorical surprisal	Valid — Definition [new in v1.1]. $K(x_B x_A)$ via I-KC. Replaces D7. Well-defined up to additive constant $c$ . Structurally invariant.
I-KC: Kolmogorov import	Import — named [new in v1.1]. $K(x y)$ and coding theorem from ZP-C. Not a bridge axiom. Introduces explicit ZP-C dependency into ZP-G.
AX-G1: Asymmetry Axiom	Axiom — intentional. Unchanged.
AX-G2: Source Asymmetry	Axiom — intentional. Unchanged.
R-BA: BA-G1 compatibility remark	Remark — [BA-G1 demoted from Bridge Axiom in v1.0]. Shannon entropy functor compatible with D7' up to $O(c)$ by coding theorem. No longer a premise of any theorem.
Proposition T1: Uniqueness of initial object	Valid — Derived. Relabelled Proposition in v1.2 (subsidiary uniqueness result). Unchanged. ✓

Component	Status / Notes
Lemma T2: Universal constituent	Valid — Derived. Relabelled Lemma in v1.2 (stepping-stone result). Unchanged. ✓
Lemma T3: Unreachability of 0	Valid — Derived. Relabelled Lemma in v1.2 (stepping-stone result). Unchanged. ✓
R1: Structural inversion	Valid — Remark. Unchanged.
R2: Categorical expression of self-containment	Valid — Remark [new in v1.3]. Connects T2 + AX-G2 to ZP-A CC-2 ( $\perp = \{\perp\}$ ). No new derivation; explanatory bridge note.
Proposition T4: Chains are forward-only	Valid — Derived. Relabelled Proposition in v1.2. Unchanged. ✓
Proposition T5: Functors preserve initial objects	Valid — Conditional on ZP-H T-H1 (closed). Relabelled Proposition in v1.2. Unchanged. ✓
Lemma T6-a: Identity surprisal is zero	Valid — Derived [new in v1.1]. Relabelled Lemma in v1.2. $K(x_A   x_A) = 0$ up to c. ✓
Lemma T6-b: Non-negative outward surprisal	Valid (PDF-level) — Derived from D5, D7', AX-G2. $K \geq 0$ ; strict inequality for distinct objects. NOT Lean-verified: Lean proof reduces to <code>Nat.zero_le_</code> (trivially true for any $\mathbb{N}$ -valued function; verifies nothing about $K$ ).
Proposition T6-c: Surprisal accumulates along chains	Valid (PDF-level) — Derived from D5, T6-b, subadditivity of $K$ . NOT Lean-verified: Lean proof reduces to <code>Nat.zero_le_</code> . Subadditivity of $K$ is a standard AIT result but requires $K$ -formalization absent from Mathlib.
Theorem T6: Informational singularity	Valid — Derived [rebuilt in v1.1]. Does not depend on BA-G1. Part II: undefined domain (AX-G2) — fully Lean-verified. Parts I, III: accumulation via T6-b, T6-c — PDF-level only (T6-b/T6-c not Lean-verified).
Theorem T7: Categorical Zero Paradox	Valid — Derived [Part IV strengthened in v1.1]. All six parts derived. BA-G1 not a dependency. ✓
OQ-G1: Native surprisal derivation	Closed — D7', T6. No bridge axiom remains as a theorem premise.

Zero Paradox ZP-G: Category Theory | Version 1.6 | May 2026 | Supersedes v1.4 | T6-b and T6-c: PDF-level only; Lean proofs verify non-negativity by type only (`Nat.zero_le_`), not  $K$ -theoretic content | T6 Part II: Lean-verified | Internal Working Document